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A note on the conservation of the volume flux in free turbulence

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The equation usually used to describe the conservation of volume flux in free turbulence (such as mixing layers, jets, etc.) is discussed.

Analysis

A basic property of jets is that they induce a flow towards themselves. The streamlines of the induced flow depend on whether the jet is plane or axisymmetric, and on the proximity of solid boundaries (see Taylor 1958). For example, figure 1 indicates the time-averaged flow configuration in a plane turbulent jet in free space. The two lines $y = \pm B(x)$ are the interfaces which appear in a long-time-exposure photograph of a dyed jet. Clearly, these lines bound the region of turbulent jet mixing.

The equation of conservation of the volume flux, obtained by integrating the continuity equation

$$\partial\overline{u}/\partial x + \partial\overline{v}/\partial y = 0$$

across the jet, is written as

$$d\mu(x)/dx = -2\overline{v}(x, B(x)) + 2(dB(x)/dx)\,\overline{u}(x, B(x)), \tag{1}$$

where

$$\mu(x) = \int_{-B(x)}^{B(x)} \overline{u}(x,y)\,dy$$

is the volume flux crossing a plane perpendicular to the jet axis. Since $\overline{u} = +\partial\psi/\partial y$ and $\overline{v} = -\partial\psi/\partial x$, (1) becomes

$$d\mu = 2\,d\psi, \tag{2}$$

which of course could be written down directly by inspection from figure 1. Equation (2) is a precise formulation of the volume-flux conservation in a turbulent plane jet. In contrast, the usual expression

$$\frac{d}{dx}\int_{-\infty}^{\infty} \overline{u}(x,y)\,dy = -2\lim_{y\to\infty} \overline{v}(x,y) \tag{3}$$

is ill formulated and misleading, for two basic reasons. First, the integral

$$\int_{-\infty}^{\infty} \overline{u}(x,y)\,dy$$

does not represent the *jet* volume flux that it is supposed to represent. Second, the increment in volume flux between x and $x+dx$ is not equal, in general, to $\lim_{y\to\infty}\overline{v}(x,y)$. Strictly speaking, for plane flows

$$\int_{-\infty}^{\infty} \overline{u}(x,y)\,dy = \mu(x) + 2[\psi(x,\infty) - \psi(x, B(x))].$$

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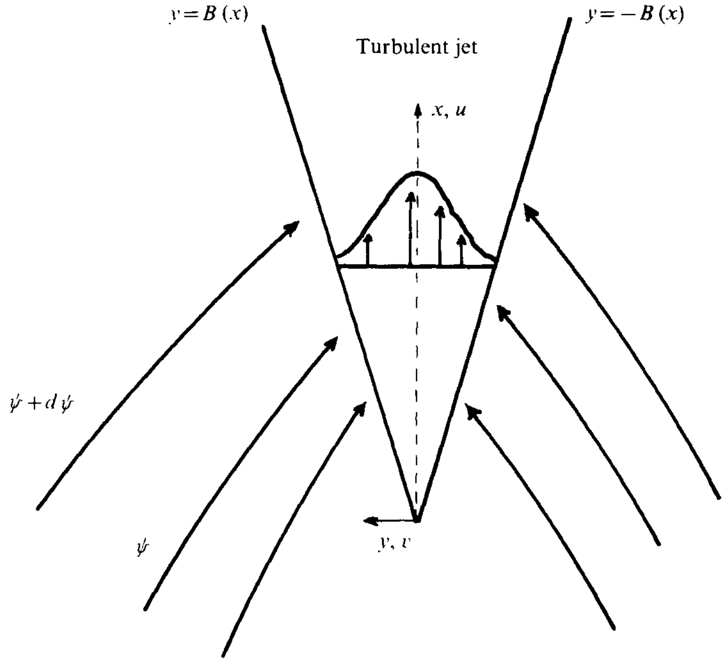

FIGURE 1. Entrainment into a plane jet in free space.

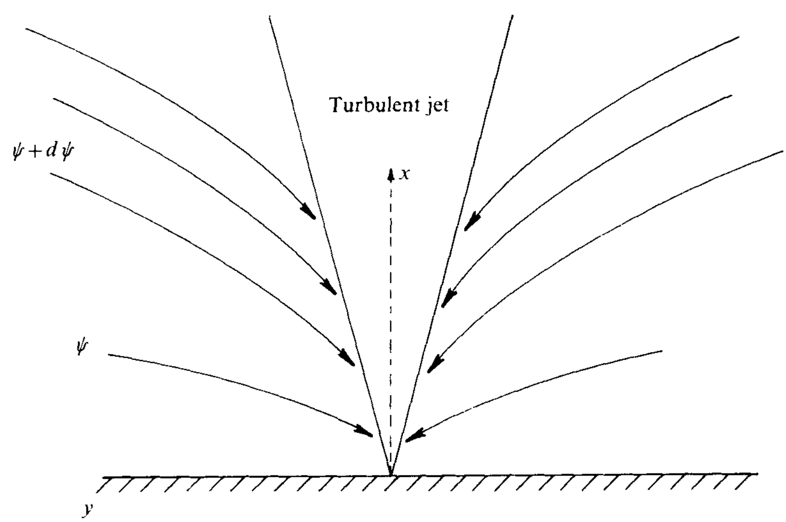

FIGURE 2. Entrainment into a plane jet out of a wall.

Using (2), we find that for a flow like that in figure 1

$$\int_{-\infty}^{\infty} \overline{u}(x, y)\, dy = \mu(\infty) = \infty.$$

In general, for any jet in free space, the integral of the axial velocity from minus infinity to plus infinity is equal to the jet volume flux at $x = \infty$, which is, ideally, infinite. The importance of solid boundaries is illustrated by the examples shown in

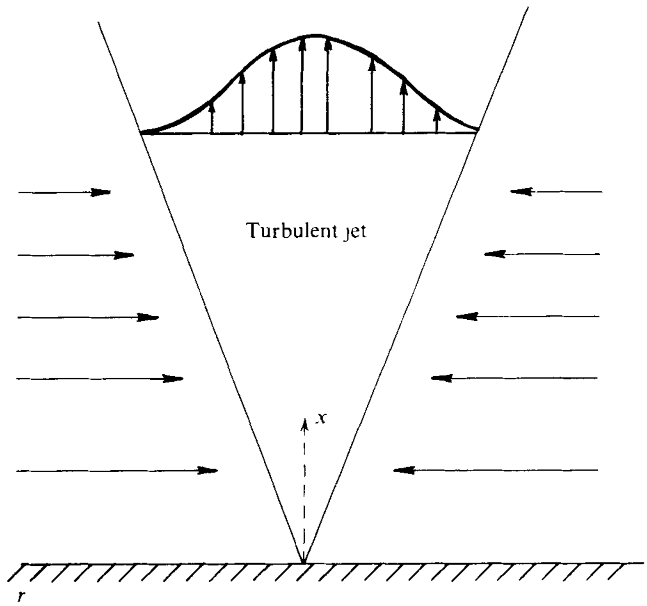

FIGURE 3. Entrainment into a round jet out of a wall.

figures 2 and 3. Figure 2 represents a line jet (momentum only) out of a wall. In this case $\psi(x,\infty) = \psi(0, B(0)) = 0$, so that

$$\int_{-\infty}^{\infty} \bar{u}(x,y)\,dy = 0 \quad \text{for any } x.$$

Apparently, by changing the flow geometry the above integral can take any value from zero to infinity. Finally, for a round jet out of a wall (figure 3) the volume flux can be represented by (3). Equation (3) is seen to be ill formulated. This was found by Crow & Champagne (1971), for a round jet, using a more restrictive approach. They, however, did not clarify the important role of solid boundaries. Equation (1) or (2) can be viewed as rigorous formulation of the equation for volume-flux conservation in a turbulent plane jet. Apparently, this discussion can be extended to any free turbulent shear layer (e.g. mixing layers, plumes, etc.).

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